It is to be understood that these drawings are for purposes of illustrating the inventive concepts of the present invention. It will also appreciated that the same reference numerals, possibly supplemented with reference characters where appropriate, have been used throughout the figures to identify corresponding parts on different figures.

DETAILED DESCRIPTION

One prime objective of network routing and management is to maximize the network revenue through efficient use of the pathways within the network. To model the maximum revenue, the following parameters need to be defined:

G(V, A): the network graph being considered;

V: the set of vertices of G;

A: the set of arcs;

K: the set of node pairs, each of which correspond a non-zero demand;

f_k: the total flow or total data transmitting rate between node pair k;

p_k: the price of per unit data flow for node pair k.

The total revenue "R" of the network flow can be formulated as:

$$R = \sum_{k} p_k f_k$$

Or equivalently, $R = \mathbf{p} \bullet \mathbf{f}$, where **p** is the price vector and **f** is the flow vector in the K-dimensional space E^K.

For a given fixed **p**, there are many different feasible **f**s. Let $\mathbf{f}^m$ **(p)** be the flow vector which gives maximum R for the corresponding **p**. That is

$$\max_{p} R = \mathbf{p} \bullet \mathbf{f}^m(\mathbf{p})$$

For any given positive λ, the following relationship will apply:

$$\mathrm{f}^m (\mathbf{p}) = \mathrm{f}^m (\lambda\mathbf{p})$$

Then $\mathbf{f}^m$ $(\mathbf{p})$ for all possible $\mathbf{p}$ represents a curve in E^K and is called the maximum-flow-frontier (MFF). Those skilled in the art will understand that the MFF is a continuous and convex curve.

To maximize the network revenue is simply to allocate the network flow as $\mathbf{f}^m(\mathbf{p})$ for a given network and price vector. Usually, the network resources remain relatively stable, and thus the MFF is remains stable. However, the price vector $\mathbf{p}$ fluctuates dynamically. There are approximate algorithms to find $\mathbf{f}^m(\mathbf{p})$ for a given $\mathbf{p}$. However, the price vector $\mathbf{p}$ fluctuates dynamically. There are approximate algorithms to find $\mathbf{f}^m(\mathbf{p})$ on a timely basis and, correspondingly, allocation of the network flow to the computed $\mathbf{f}^m(\mathbf{p})$, becomes difficult, if not impossible, if the computation speed $\mathbf{f}^m(\mathbf{p})$ is slower than the fluctuation speed of $\mathbf{p}$. For example, when the value $\mathbf{f}^m(\mathbf{p}(t-\tau))$ is computed, the current price vector $\mathbf{p}(t)$ may be $2\mathbf{p}(t-\tau)$or $\mathbf{p}(t-\tau)/2$. The computed frontier $\mathbf{f}^m(\mathbf{p}(t-\tau))$ may give a revenue which is far from the maximum revenue. Therefore, in accordance with the invention, instead of computing the $\mathbf{f}^m(\mathbf{p})$ on-line, as with prior-art methods, the method of the invention first constructs the MFF off-line then finds the right $\mathbf{f}^m(\mathbf{p})$ through a fast method for on-line computation.

The method of the invention will be better understood by reference to the figures, and to the description below. Considering Figures 1 and 2 together, Figure 1 schematically depicts an typical data network configuration having multiple data paths

between network nodes and Figure 2 graphically illustrates an exemplary three-dimensional multi-commodity data flow among, for example, the three primary nodes A, B, & C of Figure 1. Data flows between nodes A 100 and B 110 are represented by flow F in Figure 1. Also, in Figure 1 data flows between nodes B 110 and C 120 are represented by flow F_2 and, data flows between nodes C 120 and A 100 are represented by flow F_3. In Figure 2, point f1 200 on flow F_1, point f2 210 on F_2, and point f3 220 on flow F_3 represent the maximum single commodity data flow between the respective nodes. The single commodity flow values may be determined using linear programming techniques such as disclosed by Garg and Konemann, *id*. The points f1, f2 and f3 are also known as the pivots for their respective commodities.

The parameter vectors 260, 270 and 280 represent a data flow parameter characteristic that influences the allocation of data flow among the respective nodes. For example, a data flow parameter characteristic may be the cost of transmitting data along a data path or it may be the revenue collected. The summation vector 250 represents the vector summation of the parameter vectors 260, 270 and 280.

Solving a maximum revenue flow (MRF) problem for an N-dimensional flow space yields an N-1 dimensional curve, known as the Maximum Flow Frontier (MFF), that passes through all the pivots of the flow space. For example, solving an MRF problem for the three-dimensional flow space of Figure 2 yields a two-dimensional Maximum Flow Frontier (MFF). The MFF is bounded by a plane 240 that passes through all the pivots and a surface of a cube 230 that passes through all pivots. The MFF is continuous in the area surrounded by the surface 230 and the plane 240.